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Mache Effect and Combustion Instability in Solid Rocket Motor

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A theoretical analysis of nonacoustic low-frequency instability (L^* instability) of combustion in solid rocket motors (SRM) is considered. Such an instability takes place when the timescale of the thermal inertia of a combustion wave τ_T is comparable with the timescale τ_W of pressure relaxation in the motor cavity. An original model is proposed for unsteady combustion process in SRM. This model allows the flame temperature and temperature distribution in a gas flow to fluctuate with the chamber pressure [Mache effect (Mache, H., *Die Physik der Verbrennungserscheinungen*, Veit and Co., Leipzig, Germany, 1918)]. A laminar one-dimensional gas flow (without dissipation and mixing) is assumed. The propellant unsteady combustion is described using the Zel'dovich phenomenological approach and taking into account the variation of the combustion surface temperature (Novozhilov model) as well as flame temperature [Gostintsev and Sukhanov model (Gostintsev, Y. A., Sukhanov, L. A., and Pokhil, P. P., "On the Theory of Unsteady Combustion of Solid Propellant. Stability of the Process in Semi-Closed Volume," *Journal of Applied Mechanics and Technical Physics*, Vol. 12, No. 6, 1971, pp. 65–73)]. Emphasis is on the dependence of the critical conditions of stable combustion on the temperature distribution in gas flow under variable gas pressure (Mache effect). This problem is considered in detail for SRM having an end-burning charge. The result of the analysis shows that the Mache effect can significantly extend the instability region on the plane (k, $\chi = \tau_W/\tau_T$), if $\chi < 2$. Critical values of the temperature sensitivity of the combustion rate k at $\chi > 2$ are close to the asymptote (at $\tau_W \gg \tau_T$) that corresponds to the criterion of intrinsic instability of combustion under constant pressure. Application of these results to rocket motors with a common configuration of the propellant charge is discussed.

Nomenclature

A = nozzle discharge coefficient, Eq. (1), s/m
c = speed of sound, m/s
dm = mass of small gas portion, kg
F = cross-sectional area of combustion chamber, m^2
G = mass flux through the nozzle, kg/s
k = sensitivity of burning rate to the propellant initial temperature, Eq. (8)
L = length of the motor cavity, m
l_T = spatial scale of temperature disturbances in gas flow, m
M = average molecular weight, g/mol
n = ratio of specific heats (indicator of adiabatic law)
p = gas pressure, Pa or psi
R = universal gas constant, 8.314 kJ/(kg-mol K)
r = surface temperature sensitivity to the propellant initial temperature, Eq. (8)
S = combustion surface area, m^2
T = temperature, K
t = time, s
u = burning rate of propellant, m/s
V = gas-flow velocity, m/s
W = free volume of combustion chamber, m^3
α = flame temperature sensitivity to gas pressure
γ = propellant density, kg/m^3
Δl = thickness of heated layer, m
ε = flame temperature sensitivity to the temperature gradient Φ
η = nondimensional pressure fluctuation
θ = nondimensional temperature
κ = thermal diffusivity, m^2/s, or the Zeldovich parameter (8)
μ = surface temperature sensitivity to gas pressure, Eq. (8)
ν = steady-state burning pressure exponent, Eq. (8), or nondimensional fluctuation of burning rate
ρ = gas density, kg/m^3
σ = nozzle throat area, m^2
τ_C = gas residence time in combustion chamber, Eq. (3), s
τ_P = characteristic time of pressure pulsations, s
τ_T = timescale of thermal inertia of combustion wave, s
τ_W = timescale of pressure relaxation in the chamber, s
Φ = temperature gradient in subsurface layer of propellant, K/m
φ = nondimensional fluctuation of the temperature gradient
χ = ratio of timescales of combustion process in solid rocket motors, τ_W/τ_T
ψ = nondimensional fluctuation of the gas residence time
ω = frequency of oscillations, Hz

Subscripts

B = combustion products at the edge of the flame
g = gas
p = gas pressure
s = propellant surface
T = temperature distribution
W = combustion chamber
0 = steady-state or initial value
1 = amplitude of pulsation
$*$ = selected value

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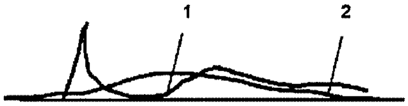
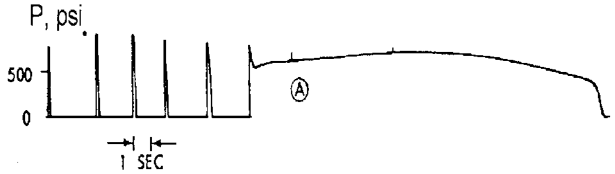
I. Introduction

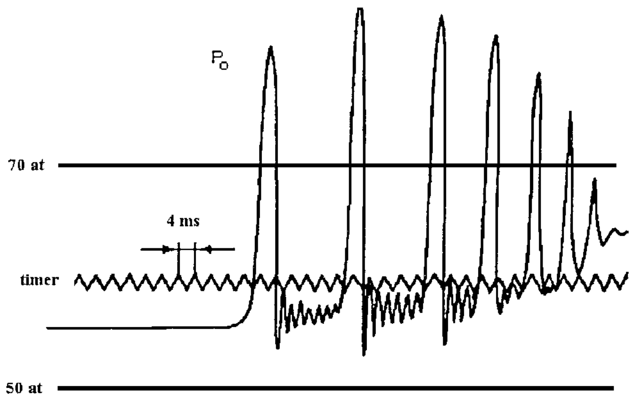
STEADY-STATE combustion in the solid rocket motor can become unstable under certain conditions.[1–10] The instability manifests itself as pressure fluctuations in a combustion chamber. A distinction is made between high-frequency and low-frequency fluctuations (see Fig. 1). The high-frequency pressure pulsations (typically having order of 1 kHz) are associated with acoustic waves propagating in the motor cavity. The acoustic waves can be significantly amplified reflecting from the combustion surface



a) The low-frequency fluctuations[1]: 1 and 2 is a pressure–time curve for subsequent attempts of combustion in one test



b) An example of chuffing[6]



c) Simultaneous existing of low and high frequency pulsations[12]

Fig. 1 Pressure fluctuations during unstable combustion in SRM.

of propellant.[3–5,10–13] The nonacoustic low-frequency instability of solid rocket motors (SRM) (L^* instability) manifests itself as chuffing or periodical pressure pulsations with frequency not more than 10 Hz (Refs. 1–7).

Starting from the pioneering works of Zel'dovich[1] and Leipunskiy,[2] L^* instability has been studied for many years (for example, see Refs. 4–7, 9, and 14–18). Despite the great importance of this problem for motor design, there is no commonly accepted quantitative theory of this phenomenon, except for qualitative interpretations of the process. The analysis in Ref. 1 shows that L^* instability takes place when the timescale of thermal inertia of a combustion wave $\tau_T = \kappa/r^2$ (κ and r are the thermal diffusivity and combustion rate of propellant) is comparable with the timescale τ_W of the pressure relaxation in the combustion chamber:

$$\tau_W = MW/A\sigma RT_B$$

where M is the average molecular weight of the combustion products and T_B is the combustion products' temperature.

It is of particular interest in SRM design to know the critical conditions for combustion stability and how they correlate with the characteristics of the propellant and the motor cavity. That is why the study of combustion instability in SRM is of traditional interest for combustion theory (for example, see Refs. 4, 5, and 10).

The majority of theories of the L^* instability assume a uniform distribution of gas temperature in the combustion chamber (and even its constancy in time). This assumption essentially simplifies the mathematical problem. However, it is well known that during adiabatic gas compression or expansion the gas temperature depends on the initial and current values of pressure. Therefore, if the pressure changes sufficiently fast, gas portions generated at different moments (different initial pressures) have different current temperatures (Mache effect).[5,19,20] As a result, there is a nonuniform temperature distribution in gas flowing from the combustion surface to the nozzle. The gas temperature fluctuations in the nozzle disturb the exhaust performance of the motor and can induce instability of the combustion process.

Reference 8 contains a significant contribution to understanding of the mechanism of gas temperature fluctuations and their influence on the L^* combustion instability in SRM.[9]

This paper is a further development of the theory in Ref. 9 in two directions. First, this study uses a quite common phenomenological description of the propellant unsteady combustion,[1,4,15] instead of any special flame model. It reflects the existence in the literature of two approaches to the problem of propellant unsteady combustion.[4,5,10] Second, this study uses the Lagrangian approach to describe the gas flow in the chamber. It allows taking into consideration the variation of gas residence time in the chamber, instead of its consideration as a constant parameter of the process. The emphasis is on the critical relationship between the propellant and motor characteristics and how it depends on the dynamics of the gas temperature fluctuations.

II. Model of the Process

Basic Assumptions

To analyze the role of Mache effect in SRM instability, we consider end-burning rocket motors. More exactly, we assume that all parts of the combustion surface of charge are equidistant from the nozzle (Fig. 2). In addition we assume the following.

1) The gas flow in the motor chamber is laminar; all gas particles formed simultaneously on the combustion surface reach the nozzle simultaneously; gas particles formed at different moments cannot be mixed; and the movement of all gas particles from the combustion surface to the nozzle is one dimensional.

2) The gas pressure distribution in the motor chamber is uniform and varies only with time: $p = p(t)$; the mass flux through the nozzle $G = Ap\sigma$ varies according to current values of pressure $p(t)$ and temperature $T(t)$ of the gas portion reaching the nozzle at the moment t; the nozzle throat area σ is constant in time.

The assumption on uniformity of pressure distribution is valid for low-frequency pressure pulsations, when the characteristic time of pressure fluctuations τ_P is much longer than timescale of the sound wave circulation in the motor cavity,[5] that is,

$$(L/c)/\tau_P \ll 1$$

where L is the length of combustion chamber.

The temperature and density fields in the gas flow are nonuniform, despite a uniform field of pressure, inasmuch as temperatures of gas portions (Lagrangian particles) formed at different moments will differ from each other due to the Mache effect. The heat exchange between neighboring gas portions will smooth the nonuniform temperature distribution (due to molecular thermal conductivity). It is easy to estimate this effect. The gas residence time in the combustion chamber is approximately equal to the timescale of pressure relaxation τ_W. During this time the molecular thermal conductivity will heat a layer having a thickness $\Delta l \approx (\kappa_g \tau_W)^{1/2}$, where κ_g is the gas thermal diffusivity.

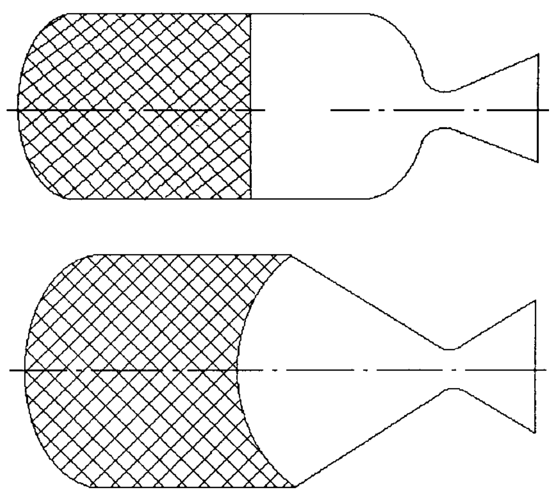


Fig. 2 Schemes of SRM with equidistant location of combustion surface from the motor nozzle.

The characteristic spatial scale l_T of the temperature disturbances in the combustion chamber, coinciding with a scale of various gas portions, is defined by the characteristic time of the pressure fluctuations τ_P and by the gas-flow velocity in the chamber V, that is, $l_T = V\tau_P = L\tau_P/\tau_W$. Under the condition

$$\frac{\Delta l}{l_T} \approx \frac{\tau_W(\kappa_g \tau_W)^{\frac{1}{2}}}{L\tau_P} \ll 1$$

a domain of intensive heat exchange between the gas particles will be a thin layer over the boundaries of the particles, and the main part of gas moves without mixture and heat exchange.

In a case of pressure oscillations in combustion chamber, the characteristic time of pulsations τ_P is inversely proportional to the oscillations' frequency ω, that is, $\tau_P \sim \omega^{-1}$. The estimations made for SRM with $L \sim 1$ m show the adiabatic approximation of gas flow is valid if the frequency ω does not exceed 100 Hz.

Formulation of the Problem

The process under consideration can be described by the system of one-dimensional equations of gas dynamics. The Euler's description of nonsteady flow[21] is traditionally used to solve the problems of interior ballistics of rocket motors. In such a case, the system of equations depends on the geometrical form of the propellant charge and the motor cavity. For example, if the charge has a spherical surface of combustion and the combustion products flow to the nozzle as to a drain in the center, the problem is described by the system taking into account the spherical symmetry. Such a system differs from the appropriate equations for a charge with flat burning surface.

A new approach is used in this paper, based on the Lagrangian description of gas flow in chamber and nozzle. Such an approach leads to the system of equations that does not depend on the specific geometry of the propellant charge.

Let a portion of combustion products be generated on the combustion surface at the moment t under pressure $p(t)$. Thus, the temperature of generated portion is equal to the flame temperature of propellant $T_B(t)$ determined by current conditions for combustion. In the framework of phenomenological theory of nonsteady combustion,[1,4,5] assuming the quasi-steady gas phase of the solid propellant, the temperature T_B can be considered as a function of instantaneous values of the pressure $p(t)$ and the temperature gradient in subsurface layer of propellant $\Phi(t)$, that is, $T_B = T_B(p, \Phi)$.

If τ_C is the gas residence time in the combustion chamber, then at the moment of the gas portion exhaust $t + \tau_C$ the pressure in the chamber is $p(t + \tau_C)$, and the gas portion temperature $T(t + \tau_C)$ is determined by $T_B(t)$, $p(t)$, and $p(t + \tau_C)$.

For the adiabatic (polytropic, in the general case) process of a Lagrangian particle movement, the temperature of a gas portion, generated at a moment t_* is equal to

$$T(t) = T_B(t_*)[p(t)/p(t_*)]^{(n-1)/n}$$

where n is an extent in adiabatic law.

Taking into account that the discharge coefficient A depends on the gas temperature at the nozzle entrance as $T^{-1/2}$, we can write

$$A(t) = A_B(t - \tau_C)\left[\frac{p(t - \tau_C)}{p(t)}\right]^{(n-1)/2n} \tag{1}$$

where $A_B(t)$ is the discharge coefficient at the gas temperature $T_B(t)$.

The volume $\mathrm{d}W$ of the gas portion generated at the moment t_* is

$$\mathrm{d}W(t) = \frac{\mathrm{d}m(t_*)}{\rho(t)} = \left[\frac{p(t_*)}{p(t)}\right]^{1/n}\frac{\mathrm{d}m(t_*)}{\rho[p(t_*)]}; \qquad \rho[p(t_*)] = \frac{p(t_*)}{RT_B(t_*)}$$

where $\mathrm{d}m(t_*)$ is the mass of the gas portion.

The total volume W of gas in the combustion chamber is equal to the integral

$$W = \int_{t-\tau_C}^{t}\left[\frac{p(t_*)}{p(t)}\right]^{1/n}\frac{\gamma S\, u(t_*)}{\rho[p(t_*)]}\,\mathrm{d}t_* \tag{2}$$

where $u(t)$ is the instantaneous value of the propellant burning rate.

As the volume of the combustion chamber is given, expression (2) should be considered as an integral equation for pressure $p(t)$.

In a case of nonsteady combustion, the gas residence time in the chamber τ_C is a function of time t. According to the mass conservation law and to the definition of τ_C, all gas generated during the combustion period from $t = 0$ up to $t - \tau_C$ will flow out through the nozzle at the moment t. This can be written as

$$\int_0^{t-\tau_C} u(t_*)\,\gamma S\,\mathrm{d}t_* = \int_0^t A(t_*)p(t_*)\sigma\,\mathrm{d}t_*$$

Differential of this equation with respect to t gives

$$\frac{\mathrm{d}\tau_C}{\mathrm{d}t} = 1 - \frac{A(t)p(t)\sigma}{u(t - \tau_C)\gamma S} \tag{3}$$

Equations system (1–3) describes the dynamics of the temperature distribution in the chamber of SRM during nonsteady-state combustion.

III. Steady-State Combustion Stability

Here we consider the unsteady combustion in the case of small deviation from the steady-state process. The mass conservation law in a steady-state regime has a form

$$u_0\gamma S = A_0 p_0 \sigma$$

where the subscript 0 indicates a steady-state value.

As follows from expression (2), the steady-state value of the gas residence time in the combustion chamber is equal to the timescale of pressure relaxation in the chamber τ_P

$$\tau_C^0 = \frac{W\rho_0}{\gamma S u_0} = \frac{W}{A_0 R T_{B0}\sigma} \equiv \tau_P$$

To simplify we will use dimensionless variables,

$$\tau = \frac{u_0^2}{\kappa}t; \qquad v = \frac{u}{u_0} - 1; \qquad \varphi = \frac{\Phi}{\Phi_0} - 1$$

$$\eta = \frac{p}{p_0} - 1; \qquad \psi = \frac{\tau_C}{\tau_C^0} - 1; \qquad \theta = \frac{T_S - T_{S0}}{T_{S0} - T_0}$$

where T_S is the temperature of the propellant surface and T_0 is the initial temperature of propellant. With these variables Eqs. (1–3) can be reduced to a system of two equations,

$$\chi\left[\frac{1}{n}\eta(\tau) - \psi(\tau)\right] = \int_{\tau-\chi}^{\tau}\left[\left(\alpha - \frac{n-1}{n}\right)\eta(\tau_*) + \varepsilon\varphi(\tau_*) + v(\tau_*)\right]\mathrm{d}\tau_* \tag{4}$$

$$\chi\frac{\mathrm{d}\psi}{\mathrm{d}\tau} = -\left[\frac{n+1}{2n}\eta(\tau) - \frac{1}{2}\left(\alpha - \frac{n-1}{n}\right)\eta(\tau - \chi) - \frac{1}{2}\varepsilon\varphi(\tau - \chi) - v(\tau - \chi)\right] \tag{5}$$

where the motor characteristic χ is the ratio of the timescales τ_W/τ_T and α and ε are the characteristics of the flame temperature sensitivity to the pressure p and to the temperature gradient Φ,

$$\alpha = \left(\frac{\partial \ln T_B}{\partial \ln p}\right)_\Phi; \qquad \varepsilon = \left(\frac{\partial \ln T_B}{\partial \ln \Phi}\right)_p$$

Substitution in Eqs. (4) and (5) of the perturbations

$$\eta(\tau) = \eta_1\exp(\Omega\tau); \qquad v(\tau) = v_1\exp(\Omega\tau)$$

$$\varphi(\tau) = \varphi_1\exp(\Omega\tau); \qquad \psi(\tau) = \psi_1\exp(\Omega\tau)$$

and exception of ψ_1 leads to

$$\left\{(1/n)\Omega\chi + (n+1)/2n - [\alpha - (n-1)/n]\left[1 - \tfrac{1}{2}\exp(-\Omega\chi)\right]\right\}$$
$$\times\,\eta_1 - \upsilon_1 - \varepsilon\left[1 - \tfrac{1}{2}\exp(-\Omega\chi)\right]\varphi_1 = 0 \qquad (6)$$

Equation (6) closes the system of equations describing the propellant unsteady combustion[4,5]

$$\frac{1+\sqrt{1+4\Omega}}{2}\theta_1 - \varphi_1 - \frac{1-\sqrt{1+4\Omega}}{2\Omega}\upsilon_1 = 0$$
$$(k+r-1)\upsilon_1 - k\varphi_1 - (\delta - \nu)\eta_1 = 0$$
$$(k+r-1)\theta_1 - r\varphi_1 - (\delta + \mu)\eta_1 = 0$$

The solvability condition of the closed system yields the characteristic equation

$$\frac{\sqrt{1+4\Omega}-1}{2\Omega}\left\{ky + \varepsilon(\delta-\nu)\left[1 - \frac{1}{2}\exp(-\Omega\chi)\right]\right\}$$
$$-(k+r-1)y + \delta - \nu - \frac{\sqrt{1+4\Omega}+1}{2\Omega}$$
$$\times\left\{\delta - ry + \varepsilon(\delta+\mu)\left[1 - \frac{1}{2}\exp(-\Omega\chi)\right]\right\} = 0$$
$$y = \frac{1}{n}\Omega\chi + \frac{n+1}{2n} - \left(\alpha - \frac{n-1}{n}\right)\left[1 - \frac{1}{2}\exp(-\Omega\chi)\right] \qquad (7)$$

where k, r, μ, and ν are the sensitivity parameters of combustion rate u_0 and surface temperature T_{s0} to variation of initial temperature T_0 and pressure,[4,5]

$$k = (T_{s0} - T_0)\frac{\mathrm{d}\ln(u_0)}{\mathrm{d}T_0}; \qquad r = \frac{\mathrm{d}T_{s0}}{\mathrm{d}T_0}$$
$$\mu = (T_{s0} - T_0)^{-1}\left(\frac{\mathrm{d}T_{s0}}{\mathrm{d}\ln p}\right); \qquad \nu = \frac{\mathrm{d}\ln(u_0)}{\mathrm{d}\ln p} \qquad (8)$$

and δ is their Jacobian,[4] $\delta = \nu r - \mu k$.

IV. Limits of Steady-State Combustion Stability

The critical conditions of combustion stability in SRM correlate with solutions of the characteristic equation (7) at imaginary values of parameter Ω (the neutral stability line). Some numerical results of the Eq. (7) solution are shown in Figs. 3 and 4. The boundary of the combustion stability region on the plane (k, χ) is presented for a propellant having the following sensitivity parameters: $r = \frac{1}{3}$, $\nu = \frac{2}{3}$, and $\mu = 0$ in Fig. 3 and $r = \frac{1}{3}$, $\nu = \frac{2}{3}$, and $\mu = 0.2$ in Fig. 4.

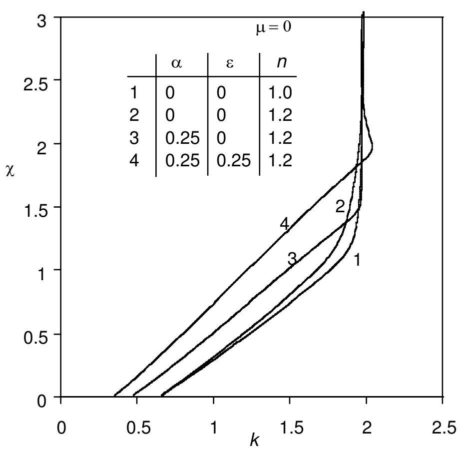


Fig. 3 Boundaries of combustion stability regions, μ = 0, r = $\frac{1}{3}$, and ν = $\frac{2}{3}$, where 1 is isothermal gas flow in the chamber (a = ε = 0, n = 1), 2 is isothermal combustion products (a = ε = 0, n = 1.2), and 3 and 4 are nonisothermal combustion products (a = 0.25, n = 1.2), for ε = 0 (3) and ε = 0.25 (4).

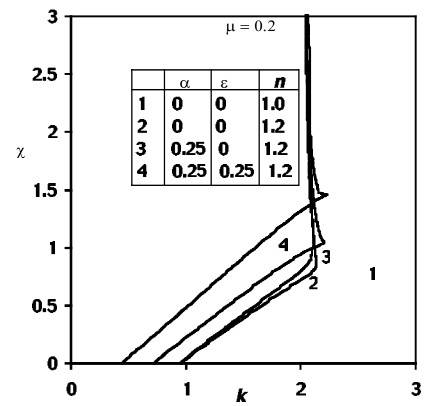


Fig. 4 Boundaries of combustion stability regions, μ = 0.2, r = $\frac{1}{3}$, ν = $\frac{2}{3}$, where 1 is isothermal gas flow in the chamber (a = ε = 0, n = 1), 2 is isothermal combustion products (a = ε = 0, n = 1.2), and 3 and 4 are nonisothermal combustion products (a = 0.25, n = 1.2), for ε = 0 (3) and ε = 0.25 (4).

Line 1 in Figs. 3 and 4 corresponds to the isothermal products of combustion ($\alpha = \varepsilon = 0$) and isothermal process in the combustion chamber of SRM ($n = 1$); the Mache effect is absent. A nonisothermal polytropic process in the combustion chamber (for example, at $n = 1.2$), even with isothermal combustion products ($\alpha = \varepsilon = 0$), leads to the narrowing of the region of combustion stability in SRM (line 2, Figs. 3 and 4). Nonisothermal products of combustion cause additional destabilization of the combustion process in SRM. The limit of stability for this case is presented by lines 3 ($\alpha = 0.25$, $\varepsilon = 0$) and lines 4 ($\alpha = 0.25$, $\varepsilon = 0.25$) in Figs. 3 and 4.

Note that all curves have a common vertical asymptote corresponding to the boundary of combustion stability at constant pressure. It is easy to see in Figs. 3 and 4 that the Mache effect leads to increase of 1.5–2 times the critical values of the motor characteristic χ at the identical values of parameter k. This result correlates with the conclusion made in Ref. 9.

Note the quite unusual behavior of the critical curve at nonzero values of parameters α and ε. For example, even at $\mu = 0$ the critical curve approaches the vertical asymptote from the side of large values of k. In this case, there is a small region of parameter χ where the appropriate range of parameters k is wider than that one at constant pressure (line 4, Figs. 3 and 4).

Such a shape of the stability region boundary is not observed in the case of isothermal combustion products and isothermal process in the chamber at $\mu = 0$ (Refs. 4 and 5). Numerical results of the analysis show the expansion of the stability region for nonisothermal processes at $\mu = 0.2$ differs from that one at $\mu = 0$. The boundary of stability at $\mu = 0.2$ has break points in this area. Close to these points the critical value of parameter k can change significantly under a small deviation of parameter χ.

V. Remarks on Critical Length of Combustion Chamber

The unexpected result is the boundary of the stability region does not depend clearly on the length of combustion chamber, but depends only on the chamber volume (more precisely, on the reduced length of the chamber $L_* = W/\sigma$), despite that one-dimensional propagation of the entropy waves in the chamber has been considered and that the critical condition of instability could be interpreted as a resonance between the entropy waves and the oscillatory system (the burning propellant).

This is a seeming contradiction. To consider a reason for it, let us suppose, for simplicity, that combustion chamber is a one-dimensional channel with constant cross section. One end of this channel coincides with the burning surface of propellant, another one adjoins to the nozzle. The gas residence time in the chamber is equal to L/V. The gas velocity V can be estimated by $V = G/\rho F$, where G is the mass flux through the nozzle (in a steady-state process it is equal to the mass flux through any cross section of the chamber) and F is the cross-sectional area of the chamber. Thus, the gas residence time is equal to $LF\rho/G = W\rho/G$. It is easy to see (taking into account $G = Ap\sigma$) that the gas residence time is equal to the characteristic time of pressure relaxation in the chamber τ_W. That is why the residence time depends only on the chamber volume and does not depend additionally on other geometrical parameters of the chamber.

This result is valid only for SRM with end-burning charges. In a case of charges having a burning channel, the gas residence time in the chamber depends on location of the gas birth place on the channel surface. That is why the combustion stability of tubular grains in SRM depends on length of combustion chamber (or the channel) along with the chamber volume.[22] It is a reason for the existence of a critical length of the channel[12,22] under the constant value of the gas volume. If the channel length exceeds the critical one, then the combustion process is accompanied by low-frequency pressure pulsations of high amplitude (see Fig. 1).

VI. Conclusion

In conclusion we underline the main result of this study. The nonuniform distribution of temperature in gas flow (Mache effect) is one of the most important factors that are responsible for combustion instability in SRM. The influence of the Mache effect on the stability boundary is comparable with influence of the initial temperature of propellant. The Mache effect can significantly (in 1.5–2 times) extend the instability region of the parameters on the plane (k, χ), if $\chi < 2$. From the viewpoint of SRM design, it forces an increase in the allowable minimum volume of the combustion chamber in 1.5–2 times to keep stable operation of SRM under control.

Acknowledgments

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